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Influence of temperature on parametric x-ray intensity

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We have measured the temperature dependence of parametric x-ray radiation (PXR) from a Si single crystal bombarded by 900-MeV electrons. The radiation intensity is appreciably enhanced when the crystal is cooled to liquid-nitrogen temperature as compared with the case for room temperature. The enhancement factors are larger for the higher-order reflections and are quantitatively consistent with the assumption that the PXR intensities are proportional to the squared Debye-Waller factor.

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It has been known for some time that well-collimated monochromatic x rays are emitted to an angle $2\theta_B$ when a crystalline target is bombarded by a high-energy charged particle with incident angle θ_B relative to a crystal plane. This phenomenon is called parametric x-ray radiation (PXR) and its characteristics have been theoretically predicted by using a kinematical approximation [1–3]. The experimental works on PXR have shown that the angular dependence and polarization nature are consistent with the theory [4–6]. On the other hand, the PXR under the strong-absorption condition is a complex phenomenon, as demonstrated by the experiment performed at the $K\alpha$ absorption edge [7], and is subject to further theoretical investigation. Another interesting point to study is the temperature dependence of the PXR intensity: Although the existing theory does not explicitly describe the temperature dependence, natural expectation would be that it is proportional to the squared Debye-Waller factor, as in the case of x-ray diffraction in the kinematical approximation

$$\exp(-2w)=\exp[-u^2(T)(2\pi n/d)^2] ,$$

where $u^2(T)$ is the mean-square amplitude of the thermal vibration at the crystal temperature T, and the interplanar distance and the order of reflection are denoted by d and n, respectively. Obviously, the higher-order reflection is expected to be the more affected by the temperature variation. Note that this temperature dependence would be altered if a dynamical effect plays an important role, as is the case for x-ray diffraction [8].

This paper presents the results of an experiment to investigate the effect of uniform cooling of a crystalline target on the PXR intensity. The experiment was carried out by using the internal beam of the Tomsk electron synchrotron, SIRIUS, operated at 900 MeV. The target was a 0.37-mm-thick monocrystalline silicon plate with the surface orientation (001) and the horizontal edge in parallel to the [110] crystallographic axis. It was mounted on a goniometer that could cool the crystal down to 88 K by means of forced pumping of liquid nitrogen flowing through the crystal holder. The temperature was controlled by a set of calibrated gauges attached to the holder at the closest point to the crystal. The stability of the crystal temperature was within 2 K for any operating temperature between 80 and 300 K. The target was oriented to let the electron beam intersect the (110) plane at the angles $\theta_B \simeq 45°$. At the detection angle $\theta_D \simeq 90°$, about 2 m from the crystal, we placed a proportional counter filled with 90% Xe and 10% CH_4. Its entrance window covered the angular widths 3.7×15 mrad^2. The energy resolution and the detection efficiency of the counter were 20% and 80%, respectively, for the Cu $K\alpha$ line.

In order to prevent the multitraversals of the electrons across the internal crystal target, we adjusted the position of the beam scraper in the synchrotron ring such that the total energy of the bremsstrahlung, measured by the quantameter, was equal to the theoretical value for the given target thickness and the beam current. During the PXR measurements, the electron-beam intensity was reduced to less than 10^8 s^{-1} to make the dead time of the counter negligible. In order to stably monitor the beam current under such low-intensity operation, we continuously measured the x-ray intensity from the synchrotron radiation emitted at the bending magnet of the accelerator ring.

A typical set of x-ray spectra obtained is shown in Fig. 1, where solid and dashed lines represent, respectively, the data for $T=88$ K and $T=293$ K. The dot-dashed curve is the background spectrum obtained with disoriented crystal at $T=293$ K. We found that the temperature dependence of the background spectrum is less than the normalization error, i.e., 5%. These data were obtained for the same number of incident electrons, 4.5×10^{11}. Comparing the peaks in Fig. 1 with the expected PXR energies shown in the second column of Table I, we notice that there is a clear $n=4$ peak, but that the $n=2$ peak is much suppressed by the detector inefficiency and the absorption by air and the vacuum chamber window. It is also noticed that the spectral shape is much distorted and looks like a double peak in the energy region corresponding to $n=6$ reflection. We

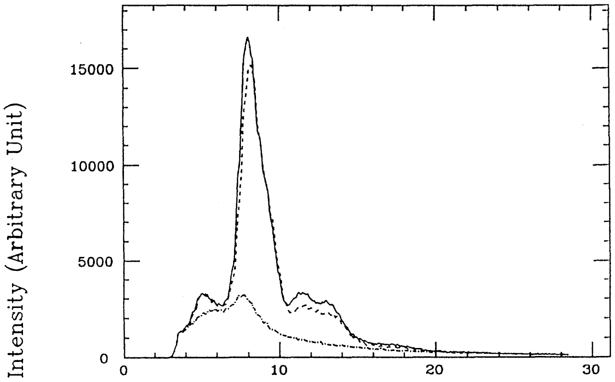

FIG. 1. Energy spectra measured by the x-ray detector at $\theta_D=90°$. The solid and the dashed curves were obtained by making the (110) plane of the Si crystal satisfy the Bragg condition, $\theta_B=45°$, at crystal temperatures $T=88$ and 293 K, respectively. The dot-dashed curve shows the background spectrum obtained by disorienting the crystal.

found that these peaks reduced to a broad single peak when we slightly deviated the crystal angle from θ_B. So, we regarded both of these peaks as belonging to the $n=6$ reflection, though the spectral shape is not fully understood yet.

We have obtained a number of spectra by rotating the crystal by 0.73-mrad steps around the vertical axis to survey the orientation dependence near θ_B with the narrowed detector aperture 2.34×15 mrad^2 After subtracting the background spectrum, these spectra have been fitted by five Gaussian functions: two Gaussian functions representing $n=4$, another two Gaussian functions for $n=6$, and the remaining one for $n=8$. The radiation intensity, defined by the area under these fitted curves, is shown in Fig. 2 for $n=4$, 6, and 8 as a function of the crystal rotation angle. The PXR intensities for warm (cool) crystal, denoted by I_w (I_c), are defined as the integrated intensity within the angular region as indicated by arrows in Fig. 2 for $T=293$ K ($T=88$ K). The ratios of the intensities $R(n)=I_c/I_w$ obtained for each order of reflection n are listed in Table I. The quoted errors are due to statistical and fitting errors only. We notice that the higher-order reflections are the more enhanced at low temperature, as expected. The reflection order, n, dependence of $\sqrt{\ln R(n)}$ is shown in Fig. 3. We see that it is consistent with the linear dependence on n. By fitting the experimental data with a line,

$$\sqrt{\ln R(n)}=(2\pi/d)n\sqrt{\Delta}\ ,$$

with $d=0.543\,07$ nm, we determined the value of Δ to be $(3.26\pm0.26)\times10^{-5}$ nm^2.

The theoretical value of the Debye-Waller factor is obtained by using the standard relation between the Debye

TABLE I. The experimentally determined ratios of the PXR intensities at $T=88$ and 293 K are shown in the third column. The first and the second columns indicate the order of reflection and the expected photon energy in keV, respectively.

n	$h\nu$ (keV)	$R(n)$
2	4.6	
4	9.2	1.16±0.03
6	13.8	1.33±0.04
8	18.4	2.10±0.27

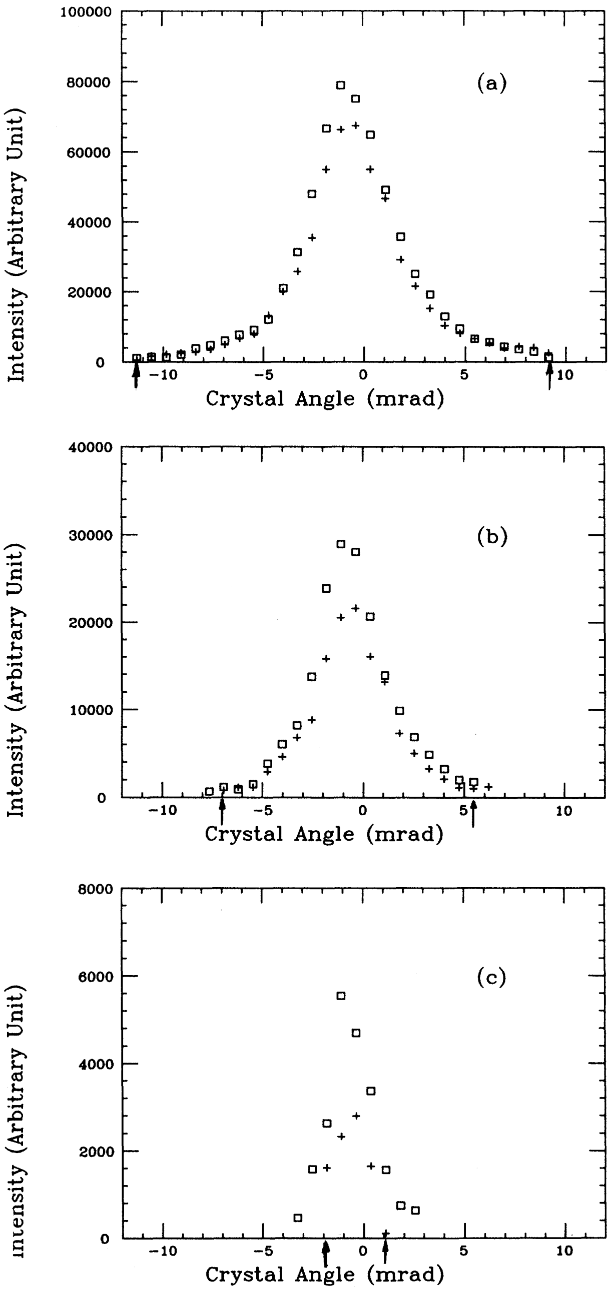

FIG. 2. Angular dependence of the radiation intensity for $n=4$, 6, and 8 are shown in (a), (b), and (c), respectively. The ordinates are the crystal angle measured from a standard angle, which is approximately equal to $\theta_B=45°$. The data points for crystal temperature $T=88$ and 293 K are shown by squares and crosses, respectively. The integration limits used to calculate the intensity ratio $R(n)$, as defined in the text, are shown by the arrows.

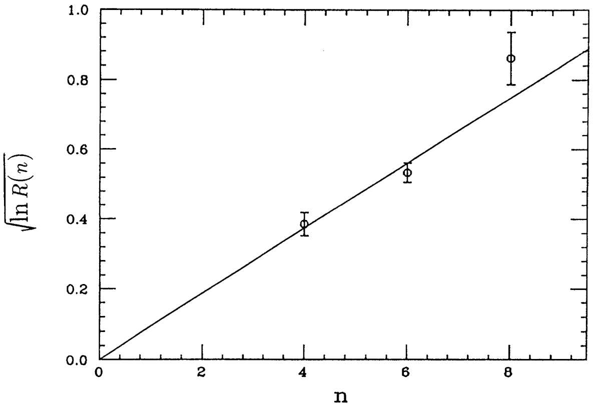

FIG. 3. Square root of the logarithm of R, the ratio of PXR intensity for cool and warm crystal, is shown as a function of n, the order of reflection. The experimental values obtained here are shown by circles. The error bars shown are due to statistical effects only. The solid line is the best fit to the data assuming that $\sqrt{\ln R(n)}$ is proportional to n.

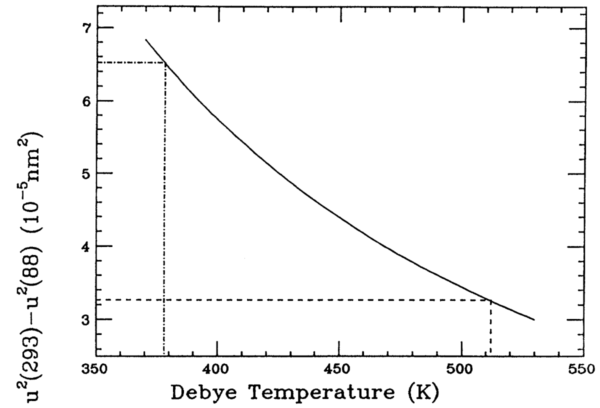

FIG. 4. Difference of mean-square amplitude of thermal vibration at $T=293$- and 88 K calculated under the Debye model is plotted against the Debye temperature T_D. The values corresponding to the assumption that the PXR intensity is proportional to $\exp(-2x)$ and $\exp(-w)$ are indicated by the dashed line and the dot-dashed line, respectively; see text for details.

temperature T_D and the mean-square vibration amplitude [9]

$$u^2(T)=\frac{3\hbar^2}{4Mk_BT_D}\left[1+4\left[\frac{T}{T_D}\right]^2\int_0^{T_D/T}\frac{y\,dy}{e^y-1}\right],$$

where $\hbar$ and k_B are the Planck constant divided by 2π and the Boltzmann constant, respectively, while the mass of the Si atom is denoted by M. The calculated difference in mean-square amplitude $u^2(293)-u^2(88)$ is shown in Fig. 4 as a function of the Debye temperature. Assuming that the PXR intensity is proportional to the squared Debye-Waller factor $\exp(-2w)$, we equate $\Delta=u^2(293)-u^2(88)$, and obtain the Debye temperature

$$T_D=512\pm18\ K\ .$$

This value is quite reasonable and lies in between the ones determined by the channeling radiation [10], $T_D=495\pm10$ K, and the ones determined by the x-ray diffraction [11], $T_D=543\pm8$ K. Note that the assumption that the PXR intensity is proportional to the Debye-Waller factor $\exp(-w)$, rather than $\exp(-2w)$, would result in an unreasonably small value $T_D=378\pm13$ K.

Thus, we may conclude that the present experimental data support the naive expectation that the PXR intensity is proportional to the squared Debye-Waller factor.

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